Comparison between high-order velocity vector and temperature structure functions

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A previously established similarity between the second-order temperature structure function, $\langle(\delta\theta)^2\rangle$, and the sum of the second-order velocity structure functions, $\langle(\delta q)^2\rangle$, is extended to higher-order moments. Measurements in a low Reynolds number turbulent wake indicate that $\langle|\delta\theta^*|^n\rangle$, for $n=2$ to 6, is in much closer agreement with $\langle|\delta q^*|^n\rangle$ than with $\langle|\delta u^*|^n\rangle$, where u is the longitudinal velocity fluctuation and asterisks denote normalization by Kolmogorov scales. The behavior of $\langle|\delta q^*|^4\rangle$ and $\langle|\delta\theta^*|^4\rangle$ at small separations is consistent with isotropy. [S1063-651X(98)12802-7]

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In a turbulent flow, the instantaneous scalar α is, primarily, advected by the instantaneous velocity vector [1,2]. The instantaneous velocity fluctuation vector $\vec{q}$ is defined as $(\vec{i}u+\vec{j}v+\vec{k}w)$, where u, v, and w are the velocity components, and $\vec{i}$, $\vec{j}$, and $\vec{k}$ are unit vectors in the x, y, z directions, respectively. A spectral analogy was proposed [3,4] between $\phi_\theta(f)$, the spectrum of the temperature fluctuation θ (f is the frequency), and $\phi_q(f)$, a spectrum defined by

$$\phi_q(f)=\frac{\langle u^2\rangle}{\langle q^2\rangle}\,\phi_u(f)+\frac{\langle v^2\rangle}{\langle q^2\rangle}\,\phi_v(f)+\frac{\langle w^2\rangle}{\langle q^2\rangle}\,\phi_w(f),$$

i.e., the sum of the weighted spectra of u, v, and w. $\langle q^2\rangle$ represents the average turbulent energy $\equiv\langle u^2\rangle+\langle v^2\rangle+\langle w^2\rangle$, angular brackets denoting averaging with respect to time. The integrals $\int_0^\infty\phi_\theta(f)df$ and $\int_0^\infty\phi_q(f)df$ are equal to $\langle\theta^2\rangle$ and $\langle q^2\rangle$, respectively. Both $\langle\theta^2\rangle$ and $\langle q^2\rangle$ are invariant with respect to space transformations. The similarity between ϕ_θ and ϕ_q was shown to be significantly superior to that between ϕ_θ and ϕ_u. In a previous paper [5], the second-order scalar temperature structure function, defined as $\langle(\delta\theta)^2\rangle$, was compared with the sum of the second-order velocity structure functions, $\langle(\delta u)^2+(\delta v)^2+(\delta w)^2\rangle\equiv\langle|\delta q|^2\rangle$. Both $\langle(\delta\theta)^2\rangle$ and $\langle|\delta q|^2\rangle$ are scalar quantities and it was argued that this comparison is more appropriate than that between $\langle(\delta\theta^2\rangle$ and $\langle(\delta u)^2\rangle$, the commonly used longitudinal velocity structure function. Also, Antonia *et al.* [6] showed that, when the molecular Prandtl number is 1, the transport equation for $|\delta q|^2$ is closely analogous to Yaglom's [7] equation, which describes the transport of $(\delta\theta)^2$. Here, we extend the analogy between $\langle(\delta\theta)^2\rangle$ and $\langle|\delta q|^2\rangle$ to higher-order moments. This should be of interest in the context of comparing the small-scale intermittencies of the velocity and temperature fields. To date, the intermittency characteristics of the velocity field have been based almost exclusively on u and the intermittency exponents inferred from $\langle(\delta u)^n\rangle$ have been compared with those inferred from $\langle(\delta\theta)^n\rangle$, e.g., [8–10]. This comparison is not altogether appropriate given that u is one component of $\vec{q}$ and θ is a scalar; also, as noted above, θ is more likely to be transported by the instantaneous velocity vector than by just the longitudinal velocity. A more appropriate comparison would be between a scalar quantity derived from the velocity vector increment $\delta\vec{q}=\vec{q}(x+r)-\vec{q}(x)\equiv\vec{i}\,\delta u+\vec{j}\,\delta v+\vec{k}\,\delta w$ where r is the longitudinal separation. To convert to a scalar quantity, we take the dot product $\delta\vec{q}\cdot\delta\vec{q}\equiv(\delta u)^2+(\delta v)^2+(\delta w)^2\equiv|\delta q|^2$. Accordingly, we consider the nth-order absolute structure function $\langle|\delta q|^n\rangle\equiv\langle[(\delta u)^2+(\delta v)^2+(\delta w)^2]^{n/2}\rangle$ and compare it with $\langle|\delta\theta|^n\rangle$ and $\langle|\delta u|^n\rangle$. Since the focus is on values of r within the dissipative range, the Kolmogorov velocity and temperature scales (these are defined below) are appropriate for normalizing velocity and temperature increments.

Measurements were made in the self-preserving region of a turbulent wake. A small Reynolds number was used, primarily to minimize the effect of spatial resolution of the sensors; this was feasible because a relatively large value of the Kolmogorov microscale η $(\equiv\nu^{3/4}\langle\epsilon\rangle^{-1/4}$, where ν is the kinematic viscosity and $\langle\epsilon\rangle$ is the mean turbulent energy dissipation rate) was possible. Also, the local turbulent intensities are small, thus allowing the use of Taylor's hypothesis for converting temporal to spatial increments. Experimental details can be found in [5]; only a brief summary is given here. The nonisothermal wake was generated with a heated aluminum tube of external diameter $d=6.35$ mm at a free-stream velocity $U_\infty=3.6$ m/s; the Reynolds number $R_d=1500$ $(\equiv U_\infty d/\nu)$. Measurements were made on the center line at $x/d=240$ where R_λ $(\equiv u'\lambda/\nu)$ is 40. η is 0.64 mm and the Kolmogorov velocity scale U_κ $(\equiv\nu^{1/4}\langle\epsilon\rangle^{1/4})$ is 0.024 m/s. $\langle\epsilon\rangle$ is estimated using the isotropic relation $\langle\epsilon\rangle=15\nu\langle(\partial u/\partial x)^2\rangle$. The mean temperature excess T_0 is 0.4 °C and the Kolmogorov temperature scale θ_κ $[\equiv\langle\epsilon_\theta\rangle^{1/2}(\nu/\langle\epsilon\rangle)^{1/4}]$ is 0.01 °C. $\langle\epsilon_\theta\rangle$ is the average temperature dissipation rate and is estimated from the isotropic relation $\langle\epsilon_\theta\rangle=3\kappa\langle(\partial\theta/\partial x)^2\rangle$, where κ is the thermal diffusivity. The spatial derivatives, $(\partial u/\partial x)$ and $(\partial\theta/\partial x)$, are calculated by converting the respective time derivatives using Taylor's hypothesis $(\partial/\partial x=-\overline{U}^{-1}\partial/\partial t)$. This hypothesis is also used to convert temporal increments into spatial increments. The velocity fluctuations were measured with an X probe (placed first in the x-y plane for measuring u and v and then rotated through 90° for measuring u and w). Wollaston (Pt-10% Rh) wires of 2.5 μm diameter were used and the inclined wires were 1.2 mm apart. The wire lengths were etched to 0.5 mm length and operated with constant tempera-

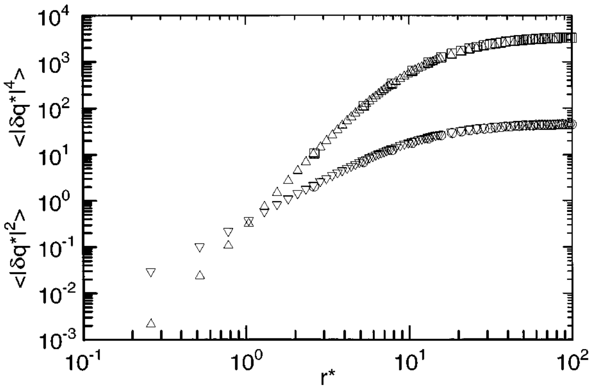

FIG. 1. Kolmogorov-normalized second and fourth-order moments of $|\delta q|$ obtained with the single X-probe method and the vorticity probe. $\langle|\delta q^*|^2\rangle$: ∇, single X probe; $\bigcirc$, vorticity probe. $\langle|\delta q^*|^4\rangle$: $\triangle$, single X-probe; $\square$, vorticity probe.

ture anemometers at an overheat ratio of 1.5. The temperature fluctuation measurements were made with a 0.63 μm diameter Wollaston wire, etched to a length of 0.7 mm. The cold wire was operated in a constant current circuit supplying 0.1 mA and its sensitivity to velocity is negligible. Appropriate gains and offset voltages were applied; all signals are filtered and sampled directly into a IBM-compatible PC using a 12-bit A/D board. The sampling frequency was set to 20 kHz. The cutoff frequency was 800 Hz for the velocity and 630 Hz for the temperature data.

By definition, simultaneous measurements of u, v, and w are required to form $\langle|\delta q|^n\rangle$ for $n<2$. The previous single X-probe measurements were not simultaneous. Simultaneous measurements were made, albeit with poorer spatial resolution with an eight wire vorticity probe operating under identical experimental conditions. The vorticity probe, described in more detail in Zhu and Antonia [11], consisted of four orthogonal X probes in a box formation of span 2η and height 2η. Although the resolution of the measurement was coarse (with $f_s=2$ kHz, the smallest step for r^* is 2.6; quantities with asterisks denote normalization by η, U_K, or θ_K), the agreement with the single X-probe values of $\langle|\delta q^n|\rangle$ is quite good (Fig. 1) for $n=2$ and $n=4$. Note that for $n=4$,

$$\langle(|\delta q|)^4\rangle=\langle(\delta u)^4\rangle+\langle(\delta v)^4\rangle+\langle(\delta w)^4+2\langle(\delta u)^2(\delta v)^2\rangle$$
$$+2\langle(\delta u)^2(\delta w)^2\rangle+2\langle(\delta v)^2(\delta w)^2\rangle. \qquad (1)$$

With the single X probe, all terms on the right side of Eq. (1) can be obtained except for $\langle(\delta v)^2(\delta w)^2\rangle$. This latter term was estimated via the isotropic relation $\langle(\delta v)^2(\delta w)^2\rangle=\langle(\delta v)^4\rangle/3$. The vorticity probe data indicated that this relation is accurate ($\pm 2\%$) for $r^*\lesssim 50$. Figure 1 suggests that the single X-probe estimates of $\langle(\delta q)^4\rangle$ should be reliable for values of r^* smaller than the Kolmogorov scale.

Figure 2 shows even-order moments of $\langle|\delta q^*|^n\rangle$, $\langle|\delta u^*|^n\rangle$, and $\langle|\delta\theta^*|^n\rangle$ for $n=2$, 4, and 6; vorticity probe data were used for $\langle|\delta q^*|^6\rangle$. The isotropic values of $\langle\epsilon\rangle$ and $\langle\epsilon_\theta\rangle$ are 20% and 12% higher than those inferred from the "4/5" and "4/3" laws, viz., $\langle(\delta u)^3\rangle=-4/5\langle\epsilon\rangle r$ and $\langle(\delta u)(\delta\theta)^2\rangle=-4/3\langle\epsilon_\theta\rangle r$. Allowing for these differences, and associated errors for U_κ, θ_κ, and η, Fig. 2 demonstrates

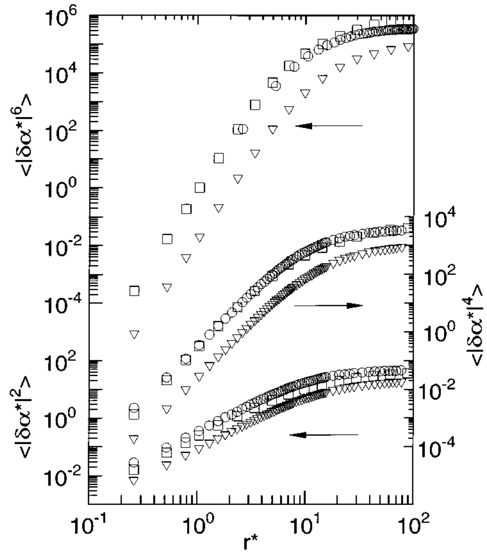

FIG. 2. Kolmogorov-normalized second, fourth, and sixth-order moments of $|\delta q|$, $|\delta\theta|$, and $|\delta u|$ as a function of r^*. $\bigcirc$, $\langle|\delta q^*|^n\rangle$; $\square$, $\langle|\delta\theta^*|^n\rangle$; ∇, $\langle|\delta u^*|^n\rangle$. Note that the origins are displaced for each n. For $n=2$ and 4, the single X-probe data are used. For $n=6$, the vorticity probe data are used.

much closer numerical agreement between $\langle|\delta q^*|^n\rangle$ and $\langle|\delta\theta^*|^n\rangle$ than between $\langle|\delta q^*|^n\rangle$ and $\langle|\delta u^*|^n\rangle$. The ratio $\langle|\delta\theta^*|^n\rangle/\langle|\delta q^*|^n\rangle$ is much closer to 1 than $\langle|\delta\theta^*|^n/\langle|\delta u^*|^n\rangle$, the latter quantity being typically 1 to 2 orders of magnitude larger than the former. Although R_λ is too small for a well-defined inertial range, the exponents $\zeta_\alpha(n)$, where $\langle|\delta\alpha|^n\rangle\sim r^{\zeta_\alpha(n)}$ (here $\alpha\equiv q$, u, or θ), were estimated using the ESS approach [12]. As expected from the distributions in Fig. 2, the magnitude ζ_q was closer to that of ζ_θ than that of ζ_u; e.g., for $n=6$, $\zeta_q\simeq 1.5$, $\zeta_\theta=1.24$, while $\zeta_u\simeq 1.78$. Clearly, it would be of interest to obtain these exponents in other flows, preferably with well defined inertial ranges. Recent observations (e.g., [13–15]) indicate that the inertial range scaling exponents of v (or w) become smaller, as the order increases, than those for u. This implies that the discrepancy [8–10] between the scaling exponents of u and θ should be reduced when considering $\langle|\delta q|^n\rangle$ and $\langle|\delta\theta|^n\rangle$.

It is of interest to compare the limiting values of $\langle|\delta q|^n\rangle$ and $\langle|\delta\theta|^n\rangle$, taking advantage of the particularly good resolution of the small scales in the present experiment. The case $n=2$ has already been considered by Antonia *et al.* [5]; the results are reproduced in Fig. 3. We will focus, primarily, on $n=4$. In the limit $r\to 0$,

$$\langle(\delta u)^n\rangle=r^n\langle(\partial u/\partial x)^n\rangle, \qquad (2)$$

with similar expressions for $\langle(\delta v)^n\rangle$, $\langle(\delta w)^n\rangle$, and $\langle(\delta\theta)^n\rangle$. For $n=2$, $\langle|\delta q|^2\rangle\equiv\langle(\delta u)^2+(\delta v)^2+(\delta w)^2\rangle$. Using Eq. (1), it follows that, for locally isotropic turbulence and in the limit $r\to 0$, $\langle|\delta q|^2\rangle=5r^2\langle(\partial u/\partial x)^2\rangle$ since $\langle(\partial v/\partial x)^2\rangle=\langle(\partial w/\partial x)^2\rangle=2\langle(\partial u/\partial x)^2\rangle$. We note, however, that the measurements indicate that $\langle(\partial v/\partial x)^2\rangle=1.9\langle(\partial u/\partial x)^2\rangle$ and $\langle(\partial w/\partial x)^2\rangle=1.8\langle(\partial u/\partial x)^2\rangle$. Such departures are within the

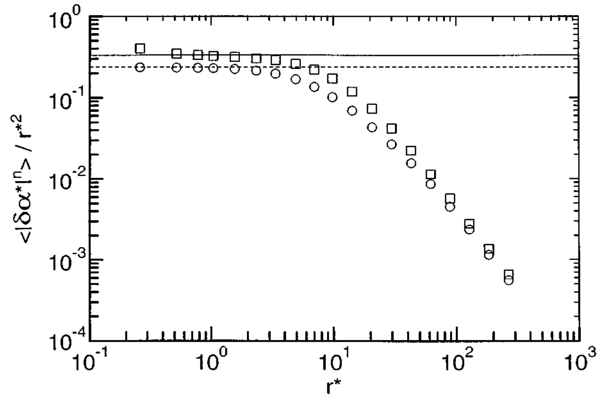

FIG. 3. Second-order moments of $|\delta q^*|$ and $|\delta\theta^*|$ multiplied by r^{*-2}. □, $\langle|\delta q^*|^2\rangle$; ○, $\langle|\delta\theta^*|^2\rangle$; —, $\langle|\delta q^*|^2\rangle r^{*-2}=1/3$, Eq. (3); ---, $\langle|\delta\theta^*|^2\rangle r^{*-2}=Pr/3$, Eq. (4).

noise level of the signals. Normalizing, with Kolmogorov scales, leads to (when $r^*\rightarrow 0$)

$$\langle(\delta q^*)^2\rangle r^{*-2}=1/3. \tag{3}$$

Similarly, in the limit $r^*\rightarrow 0$,

$$\langle|\delta\theta^*|^2\rangle r^{*-2}=Pr/3. \tag{4}$$

The experimental results in Fig. 2 ($n=2$) and Fig. 3 confirm that $|\delta\theta^*|^2$ is indeed smaller than $\langle|\delta q^*|^2\rangle$, the difference being equal to Pr.

If we now consider Eq. (1), the limiting values of $\langle(\delta u)^4\rangle$, $\langle(\delta v)^4\rangle$, and $\langle(\delta w)^4\rangle$ when $r\rightarrow 0$ are given by Eq. (2); the cross moments in Eq. (1) are given by

$$\langle(\delta u)^2(\delta v)^2\rangle=r^4\langle(\partial u/\partial x)^2(\partial v/\partial x)^2\rangle, \tag{5}$$

with similar expressions for $\langle(\delta u)^2(\delta w)^2\rangle$ and $\langle(\delta v)^2(\delta w)^2\rangle$. For isotropic turbulence, $\langle(\delta v)^4\rangle$, $\langle(\delta w)^4\rangle$, and the cross moments can be related to $\langle(\delta u)^4\rangle$ through the expression for the eighth-order correlation tensor $\langle u_{i,m}u_{j,n}u_{k,p}u_{l,q}\rangle$ [16], viz.,

$$\begin{aligned}\langle(\partial v/\partial x)^4\rangle&=\langle(\partial w/\partial x)^4\rangle=4\langle(\partial u/\partial x)^4\rangle\\ \langle(\partial u/\partial x)^2(\partial v/\partial x)^2\rangle&=\langle(\partial u/\partial x)^2(\partial w/\partial x)^2\rangle\\ &=\langle(\partial v/\partial x)^2(\partial w/\partial x)^2\rangle/2\\ &=2\langle(\partial u/\partial x)^4\rangle/3.\end{aligned} \tag{6}$$

It follows from Eqs. (1) and (5) that, in the limit $r\rightarrow 0$,

$$\langle|\delta q|^4\rangle=\frac{43}{3}\,r^4\langle(\partial u/\partial x)^4\rangle$$

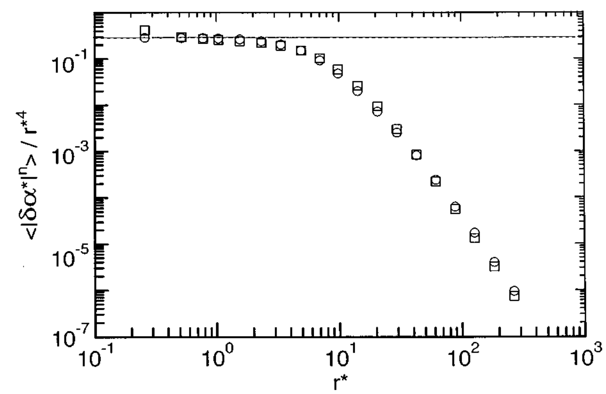

FIG. 4. Fourth-order moments of $|\delta q^*|$ and $|\delta\theta^*|$ multiplied by r^{*-4}. □, $\langle|\delta q^*|^4\rangle$; ○, $\langle|\delta\theta^*|^4\rangle$; —, $\langle|\delta q^*|^4\rangle r^{*-4}=(43/3)\,F_{\partial u/\partial x}/15^2$, Eq. (7); ---, $\langle|\delta\theta^*|^4\rangle r^{*-4}=(Pr^2/9)\,F_{\partial\theta/\partial x}$, Eq. (8).

or

$$\langle(\delta q^*)^4\rangle r^{*-4}=\frac{43}{3}\,F_{\partial u/\partial x}/15^2, \tag{7}$$

where $F_{\partial u/\partial x}$ is the flatness factor of $\partial u/\partial x$. Similarly, it is easy to show that, when $r^*\rightarrow 0$,

$$\langle|\delta\theta^*|^4\rangle r^{*-4}=\frac{Pr^2}{9}\,F_{\partial\theta/\partial x}, \tag{8}$$

where $F_{\partial\theta/\partial x}$ is the flatness factor of $\partial\theta/\partial x$. Interestingly, the coefficients of $F_{\partial u/\partial x}$ and $F_{\partial\theta/\partial x}$ are only marginally different, with the former equal to 0.064 and the latter to 0.059 (when $Pr=0.73$). Note that, when $r^*\rightarrow 0$,

$$\langle|\delta u^*|^4\rangle r^{*-4}=F_{\partial u/\partial x}/15^2,$$

i.e., a factor of 43/3 smaller than the coefficient in Eq. (7). Although Fig. 4 shows remarkable agreement between the limiting behaviors of $\langle|\delta q^*|^4\rangle$ and $\langle|\delta\theta^*|^4\rangle$ with Eqs. (7) and (8), there are individual departures from the isotropic requirements of Eq. (6), e.g., $\langle(\partial w/\partial x)^4\rangle\approx 3.7\langle(\partial u/\partial x)^4\rangle$, $\langle(\partial v/\partial x)^4\rangle\approx 4.4\langle(\partial u/\partial x)^4\rangle$, and $\langle(\partial u/\partial x)^2(\partial w/\partial x)^2\rangle \simeq 0.8\langle(\partial u/\partial x)^2\rangle$. Although these departures are not negligible, they may be of either sign; consequently, the sum of all the terms on the right of Eq. (1) approximately satisfies Eq. (7).

In summary, there is much closer agreement between $\langle|\delta\theta^*|^n\rangle$ and $\langle|\delta q^*|^n\rangle$ than between $\langle|\delta\theta^*|^n\rangle$ and $\langle|\delta u^*|^n\rangle$. In particular, for small r^*, $\langle|\delta q^*|^4\rangle$ and $\langle|\delta\theta^*|^4\rangle$ conform with isotropy. While the magnitudes of $\langle|\delta q^*|^2\rangle$ and $\langle(\delta\theta^*)^2\rangle$, in the limit $r^*\rightarrow 0$, differ by a factor equal to Pr, the magnitudes of $\langle|\delta q^*|^4\rangle$ and $\langle|\delta\theta^*|^4\rangle$ are nearly identical.

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